Representing Equality: A Tangible Balance Beam for Early Algebra Education

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ABSTRACT
In this paper we describe the design and implementation of a tangible balance beam that we created for early algebra education. We also present data from an exploratory study with seven children (ages 9-10 years) in a local elementary summer school classroom. Our results provide insight into how students solve algebra problems using our tangible interface, how they coordinate multiple representations (both digital and physical) in the problem solving process, and how they understand the concept of algebraic equality in this context. The data suggests that our interface helps students think about equations in a relational context, which has been shown to be an important skill for understanding more advanced concepts in algebra. Whether or not the combination of physical and digital representations provided by our interface helps students apply this relational understanding to equations written using standard algebraic notation is an open question that we hope to investigate in future work.

Author Keywords
Tangible interaction; early algebra education; mathematics; multiple linked representations

ACM Classification Keywords
H5.m. Information interfaces and presentation (e.g., HCI): Miscellaneous.

1. INTRODUCTION
The BEAM [9] is a tangible user interface designed to help children learn fundamental concepts of algebra. It consists of a wooden balance beam representing an algebraic equation (Figure 1). The left and right arms of the beam have positions numbered one through nine that stand for coefficients in the terms of an equation. Integers (represented with plastic tokens called pebbles) can be stacked onto each position to construct equations and inequalities. For example, to create the equation, 4 + 3 = 7, a child would first place two pebbles on the left side of the beam, one on the 3 and the other on the 4. The beam would then tilt because of the weight of the pebbles on the left side. To make the beam balance again, the child would place a final pebble on the 7 on the right side (Figure 2).

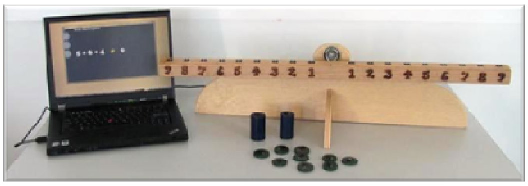

Figure 1. The BEAM. Equations are constructed on a tangible wooden beam with plastic tokens. The graphical user interface is displayed on a tethered laptop.

This process can be expanded to support more complex equations emphasizing multiplication skills. For example, one way to create the equation, $(2 \times 10) = (2 \times 2) + (4 \times 4)$, is shown in Figure 2. This equation could be rearranged to form other combinations of terms that produce the same equality, or 20 = 20.

Figure 2. A solution for the equation (2x10) = (2x2) + (4x4).

Our project builds on previous work by dynamically linking a physical balance beam with a computer display that shows the corresponding equations using standard algebraic notation (Figure 3). Our beam uses a USB cord and embedded chips to determine the state of the beam and visualize the results virtually in real time.

In this paper we present details of the beam's design and implementation. We also present data from an exploratory study with seven children (ages 9-10). Our results provide insight into how students solve algebra problems using our tangible interface, how they coordinate multiple representations (both digital and physical) in the problem solving process, and how they understand the concept of algebraic equality in this context. The data suggests that our interface helps students think about equations in a relational context, which has been shown to be an important skill for understanding more advanced concepts in algebra.

2. BACKGROUND

2.1 Balance and Equality

The metaphor of balance extends to many topics, particularly algebra education and the concept of equality. Knuth, Stephens, McNeil, and Alibali [7] have studied students' conceptions of the equal sign as an operational (i.e., "the answer is") rather than relational (i.e. "both sides have the same value") symbol. This operational understanding has been shown to be common in algebra education and to negatively affect students' performance on algebra assessments [7]. Researchers have examined the relationship between knowledge of operation patterns and difficulties with equations to find that when operational patterns are highlighted students are less likely to use the correct strategy to solve an equation [11].

These understandings and performance routines are built from two main forms of knowledge in math experiences: conceptual (highlighting the concept of equivalence) or procedural (highlighting the procedure for solving equivalence exercises) [16]. Both forms of instruction increase conceptual understanding as well as the understanding and correct use of instructed procedures; however, conceptual instruction also leads to proper transfer of procedural methods in other problems [16].

Not surprisingly, the balance beam analogy is often used to reinforce the concept of equality in early algebra education. Three prominent examples exist: the Virtual Balance Scale, a online application that displays equations using standard algebraic notation and a virtual pan balance [13]; Hands-On Equations, a physical manipulative with tokens on a static pan balance [3], and the EquaBeam, a physical balance beam that tilts as tokens are hung on coefficient positions across the beam [5]. The EquaBeam is similar to our physical beam in design, but lacks the ability to connect to a computer and dynamically link multiple representations. The work presented in this paper attempts to merge advantages of these three systems, by combining a physical balance beam with a dynamically linked computer display.

Many researchers have explored the use of tangible interfaces and technology to support mathematics learning [6, 14, 17, 18, 19]. Though many digitally enhanced manipulatives with mathematical focus have been developed, those with a focus in algebra are rare. DigiQuilt allows users to design patchwork quilt-blocks in a screen-based manipulative environment found to promote engagement and skill with fractions [8]. Quadratic is a virtual interactive tabletop manipulative for collaborative exploration of algebraic expressions making graph production an easy and engaging [15]. The work presented here attempts to build on these tools and expand tangible math education to include foundational algebraic concepts.

Research has shown mixed results in tangible education; concrete materials can help or hinder children's learning depending on the appropriateness of the material for the lesson, the structure of the environment, and the connection made between the material and the lesson [4]. Perceptually rich concrete objects often result in more errors in student work; though these errors are less likely to be conceptual [12]. In recent research, Marshall, Cheng, and Luckin [10] compared learning outcomes in adults using tasks on both a physical and a virtual balance beam; however, they found no differences in learning outcomes in the two conditions [10]. Blikstein and Wilensky [2] found the combination of physical tools with virtual environments draw attention to particular attributes, highlight procedures, and moderate cognitive load drawing focus to the primary topic.

3. THE BEAM

3.1 Physical Interface

The physical balance is constructed from a thirty-six inch wooden beam connected to a stand by means of a simple ball bearing. Digits (1–9) representing the coefficients in the terms of an equation are placed evenly towards either end of the beam. The pebbles provide the weights needed to shift the beam and may be stacked to add value to terms. We constructed the pebbles out of ABS plastic rods that we machined to shape. We selected ABS for its machinability, heft, sturdiness, and *feel*. The tens pebbles are ten times that of the unit pebbles in weight, volume, and height and colored blue to differentiate them from the green units. LEGO® Technic connectors are glued inside each pebble. They provide a stable, two-wire electrical and mechanical connection; they can be stacked to an arbitrary height; they have a low profile (approximately ¼ inch high); and they are familiar to many children. Each pebble also contains a Dallas Semiconductor 1-Wire DS2401 chip that provides a unique 48-bit serial number. These serial numbers are transferred from the pebbles to the system using a roll-call protocol established by the 1-Wire bus system.

3.2 Digital/Physical Information Transfer

An Arduino Mega board (http://arduino.cc/) serves as the connection from the physical system to the laptop computer. Each position along the beam is a separate 1-Wire bus connected to a specific input pin on the Arduino. This connection consists of LEGO® Technic connectors wired along the beam to the ground and one unique data pin on the board. The Arduino determines the position and value of all the pebbles on the beam (with a scan rate of 100ms) as the pieces are stacked on the positions, and relays this information back to the laptop computer.

3.3 Graphical User Interface

We created a Processing application (http://processing.org) to display equations from the beam using standard algebraic notation (Figure 3). As pebbles are added to the beam, their values appear by "falling" into the onscreen algebraic equation. Likewise, when the pieces are removed, they drop off of the screen and the equation shifts to occupy the space.

4. RESEARCH STUDY

To improve our understanding of how the beam might be used in schools, we conducted an exploratory study with seven children (ages 9-10) in a local elementary summer school classroom. We were interested in how students used the beam to solve problems and how they coordinated the physical and digital representations in the process.

4.1 Participants

Participants were recruited from a mixed-grade classroom (grades 2 - 4). The school primarily serves middle-income families with diverse ethnic backgrounds. Seven children (six girls and one boy, ages 9-10 years) volunteered with parental consent and were randomly assigned to three groups of two students with a remaining group of one.

4.2 Procedure

The study was conducted at a table in the back the students' classroom while class was in session. We video and audio recorded each session with the exception of the group of one, who did not wish to be recorded. Each session lasted an average of 25 minutes resulting in approximately four hours of observational data.

During the first session, we gave each participant a written pre-test with three open response questions developed by Knuth et al. [7] to asses students' understanding of the equal sign as a relational operator. After the pre-test, we asked the participants to solve a small number of equations using pencil and paper, introduced each group to the BEAM, and had them practice by constructing two or three equations on their own. In the second session, we gave students additional equations and asked them to create them on the BEAM. Two students continued for a third session, working through equations both on and off the BEAM. We also asked each student to complete a written post-test, identical to the pre-test, at the end of the final session.

5. RESULTS

5.1 Equation Strategies

Students used the beam to construct equations in much the same way that they solve written equations; most prominently, they tended to put the answer on the right side by either swapping sides physically or verbally. For example, given the equation 9 = 8 + 1, students would swap the sides of the equation physically so that the single term was on the right side of the beam (i.e. they would construct the equation with 8 + 1 on the left side and the 9 on the right side). Other students would construct the equation as it was written; that is, 9 on the left side and 8 + 1 on the right side. However, when asked what they had created they would explain, "eight plus one" (pointing the right side) "equals nine" (pointing to the left side).

More complex equations forced students to deviate from these patterns. To create any number greater than 9 using the beam, students must either add values to a coefficient, consider using a different coefficient or a combination.

5.2 Interpretation of Equal Sign

According the equality pre/posttest, there was no change in interpretation of the equal symbol for almost all of the participants over the course of the study. Although students did not express a relational understanding in the pre/post test, they did discuss their exercises with the beam in a relational context; children demonstrated an understanding that the values on the sides were or were not the same based on the beam's tilt.

5.3 Missing Values

When asked to find missing values for equations in the pencil and paper exercises, students used four strategies. Rittle-Johnson and Alibali identified three of these strategies: add all; add to equal sign; and equalize [16]. We observed one additional strategy that we call *solve and continue*. In this strategy, first the correct value for the blank is found using the Equalize method; however, the student continues to work on the equation by adding the values for both sides together. For example,

$$3 + 4 + 2 = 5 + \underline{4} \rightarrow 9 = 9 \rightarrow 18$$

Of these four, the Equalize procedure is the only process resulting in the correct answer; only one student demonstrated the "Equalize" procedure on paper. When given missing value equations using the beam all five students used the Equalize procedure. The difference between pencil and paper exercises and the beam is likely due to the real-time feedback provided by the beam, both the physical tilt of the beam itself and the onscreen = or ≠ sign. Whether or not this feedback contributes to the learning process in any way is an open question.

5.4 Multiple Representations

One goal of multiple linked representations is to provide an opportunity for students to map concepts from one representation to another, ideally reducing their cognitive load [1]. To understand how students coordinated representations with the BEAM, we analyzed our video recordings to determine student gestures and eye gaze from moment to moment. We found that the children seemed to be using the computer display for three main purposes: 1) verification of pebble placement on the intended number with a stable electrical connection, 2) checking to see that they had in fact created a balanced equation using the white or yellow = or ≠ sign, and 3) reading their final equation. Students consistently used the tilt of the beam to determine the equality of equations. When there was an error in their solution, students would validate incorrectness with the beam's tilt. However, when their answers were correct, the beam's tilt served as a secondary form of feedback, and they would validate correctness based on the equal sign displayed in the screen.

6. Study Limitations

The study reported here was exploratory in nature and involved only a small number of children interacting with the beam in two or three short sessions.

7. Conclusion & Future Work

In addition to revealing some shortcomings of our existing design, the exploratory study presented here also highlights opportunities for future research. Our results provide some evidence that students apply a relational understanding of the equal sign when solving problems using use the beam, and that they can solve problems using the beam that are more difficult with pencil and paper. An open question, therefore, is whether or not using the beam can help students develop and apply a relational understanding of equality to their written work without the beam. We hypothesize that by dynamically linking physical and digital representations, the beam will help to reinforce the metaphor of physical balance in the context of algebraic equality. This is applicable for both the physical and virtual representations (e.g. the Virtual Balance Scale); however, we hope to test whether or not a physical representation will be more effective. We also hope to explore ways in which the beam can be most effectively incorporated into existing early algebra curriculum.

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